Two models of imperfect delayed repair for a continuously monitored system subject to an accumulative deterioration

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ABSTRACT: A system subject to an accumulative deterioration and continuously monitored is considered. The system fails when its degradation level exceeds a predetermined failure threshold. At system failure, a signal is sent to the maintenance team, which arrives after a fixed delay and performs an instantaneous replacement. To prevent failures and shorten the down period, a condition-based maintenance strategy is applied and the maintenance team is preventively called when the degradation of the system exceeds a predetermined preventive threshold lower than the failure threshold, with the same fixed delay for the team's arrival otherwise. At maintenance time, an instantaneous but imperfect repair is performed. Two different imperfect repair models are considered, in the spirit of virtual age models for recurrent events. The first model assumes that the repair reduces the degradation of the system (Model I). The second model assumes that the repair reduces the age of the system (Model II). For both models, Markov renewal equations are obtained for some reliability indicators. Numerical examples are given to illustrate the analytical results and to compare both models of repair.

1 INTRODUCTION

Most of the systems suffer a physical degradation before the failure. Mathematical models that describe the process of degradation of systems play a central role to improve the reliability and the maintainability of these systems. For certain types of degradation processes, a model involving independent non-negative increments is appropriate. The gamma process (Singpurwalla 1995) is a natural model for degradation processes in which the deterioration is supposed to take place gradually over time in a sequence of tiny increments such as wear, fatigue, corrosion, crack growth, etc. Mathematically, a gamma process is a stochastic process with independent, non-negative and gamma distributed increments with a common scale parameter. Furthermore, the existence of an explicit probability distribution function of this stochastic process permits feasible mathematical developments.

For deteriorating systems, when the degradation level reaches a threshold, the system is no longer able to function satisfactorily. Since it is generally less costly to replace a system before it has failed, maintenance policies based on the system condition are usually proposed, aiming at preventing failures. Such maintenance strategies minimize the maintenance cost, improve operational safety and reduce the quantity and severity of in-service system failures, see (Bérenguer, Grall, Dieulle, & Roussignol 2003), (Grall, Bérenguer, & Dieulle 2002), (Huynh, Barros, Bérenguer, & Castro 2011) e.g.. Condition-based maintenance uses data collected through continuous monitoring, and based on the information data, different maintenance actions are programmed. The system condition after a maintenance action depends on the maintenance efficiency with two extreme cases: the system condition is the same as just before the maintenance action (minimal maintenance) and the system condition is the same as if the system were new (perfect maintenance). In practice, system condition after the maintenance actions lies between these two extreme cases (Doyen & Gaudoin 2004). In the literature several models combining imperfect maintenance and degradation processes have been proposed, see (Castanier, Bérenguer, & Grall 2003), (Newby & Barker 2006) and (Nicolai, Frenk, & Dekker 2009).

This paper shares the modelling assumptions of (Mercier & Castro 2013). So, a system subject to a cumulative deterioration modelled as a gamma process, continuously monitored and under an imperfect and deferred maintenance is analyzed. By deferred maintenance, we mean that the maintenance tasks are not performed when they should be carried out, due to a delay in the arrival of the maintenance team. The imperfect maintenance is developed under two models of repair. In the first model (Model I), repair reduces the degradation of the system accumulated from the last maintenance action. In the second model (Model II), repair reduces the system age accumulated from the last maintenance action. Both models of repair take into acount the overshoot of the gamma process and are analyzed under the theory of Markov Renewal processes. For both models of repair, different transient reliability measures are obtained in the framework of semi-regenerative processes with continuous space state. Numerical examples are given to compare both types of repair.

The paper is structured as follows. In Section 2, the formulation of the problem is showed. Section 3 develops the mathematical formulation that describes the functioning of the system under the two repair models. Section 4 is focused on the calculus of different transient reliability measures. Section 5 shows some numerical examples of the measures calculated previously and Section 6 concludes.

2 FORMULATION OF THE PROBLEM

We consider a system subject to a deterioration modelled by a gamma process $(X_t)_{t\geq 0}$, where X_t is gamma distributed $\Gamma(\alpha t, \beta)$ with probability distribution function (p.d.f.)

$$f_t(x) = \frac{\beta^{\alpha t}}{\Gamma(\alpha t)} x^{\alpha t - 1} e^{-\beta x} \mathbf{1}_{\mathbb{R}_+}(x),$$

where $\mathbf{1}_{\{\}}$ stands for the indicator function and $\alpha, \beta > 0$ and with Lévy measure given by

$$\mu(ds) = \alpha \frac{e^{-\beta s}}{s} \mathbf{1}_{\mathbb{R}_+^*}(s)\, ds. \tag{1}$$

The cumulative distribution function (c.d.f.) and survival function (s.f.) of X_t are denoted by F_t and $\bar{F}_t$ respectively.

The system fails when its degradation exceeds the level L with time to failure

$$\sigma_L = \inf(t > 0 : X_t > L).$$

At time σ_L, a signal is sent to the maintenance team which arrives at time $\sigma_L + \tau$ where τ is deterministic and instantaneously replaces the out-of-order system by an identical new one (at time $\sigma_L + \tau$). Hence, the system is unavailable from σ_L up to $\sigma_L + \tau$.

To reduce the system downtime, an alert signal is preventively sent to the maintenance team as soon as the system reaches a preventive maintenance level M ($0 \leq M \leq L$), namely at time σ_M,

$$\sigma_M = \inf(t > 0 : X_t > M).$$

At time $\sigma_M + \tau$, the maintenance team is ready to operate and tries to adjust the system. At the time of the maintenance,

- If the system is failed (that is, $\sigma_L \leq \sigma_M + \tau$), a corrective replacement is performed and the system is replaced by a new one.
- If the system is working (that is, $\sigma_L > \sigma_M + \tau$), an imperfect repair is performed with two models of repair:
  - Model I. The repair tasks remove a part ($\rho\%$) of the degradation accumulated from the last maintenance model ($0 \leq \rho \leq 1$).
  - Model II. The repair tasks remove a part ($\rho\%$) of the age accumulated from the last maintenance model ($0 \leq \rho \leq 1$).
- After the imperfect repair
  - If the degradation of the system is greater than M, the system is considered to be too degraded. A preventive replacement is performed and the system is replaced by a new one.
  - If the degradation of the system is less than M, the system goes on working.

In short, the maintenance actions for Model I and II are the following

- A corrective replacement (CR) when the system is broken at the arrival of the team maintenance.
- A preventive repair (PM) when the repair brings the degradation of the system below M.
- A preventive repair plus a preventive replacement (PM+PR) when the repair does not bring the degradation below M.

To specifically describe the two models of repair and the maintenance strategy, we shall make use of independent copies of $(X_t)_{t\geq 0}$ denoted by $\left(X_t^{(n)}\right)_{t\geq 0}$ for $n = 1, 2, \ldots$. Corresponding reaching times of threshold L (M) are denoted by $\sigma_L^{(n)}$ $\left(\sigma_M^{(n)}\right)$ respectively and let $(Y_t)_{t\geq 0}$ be the process that describes the evolution of the maintained system.

3 MATHEMATICAL FORMULATION

Let $S_1 = \sigma_M^{(1)} + \tau$ be the first maintenance time. At time S_1, the deterioration level $X_{S_1}^{(1)}$ is observed. If the system is not failed, namely if $X_{S_1}^{(1)} \leq L$, a preventive repair is performed. For Model I , the repair reduces $\rho\%$ of the degradation of the system. The degradation level after repair hence is $(1-\rho)X_{S_1}^{(1)}$ (deterministic function of $X_{S_1}^{(1)}$). For Model II, the repair reduces $\rho\%$ of the system age. The degradation level after repair hence is random, identically distributed as $X_{(1-\rho)S_1}^{(1)}$.

In the following, we set:

$$Z_{U_n}^{(n)} = \begin{cases} (1-\rho)X_{U_n}^{(n)} \text{ for Model I,} \\ X_{(1-\rho)U_n}^{(n)} \text{ for Model II.} \end{cases}$$

The general evolution of the maintained system for both models of repair is the following, with S_1, ..., S_n, ... the maintenance times and U_1, ..., U_n, ... the inter-maintenance times (and $U_1 = S_1$).

At time S_1 :

- If $X_{S_1}^{(1)} > L$ a corrective replacement is performed at time S_1, $Y_{S_1} = 0$.
- If $X_{S_1}^{(1)} \leq L$ a preventive repair is performed at time S_1 putting the system back to the deterioration level $Z_{S_1}^{(1)}$.
  - If $Z_{S_1}^{(1)} \geq M$, the system is unmaintainable $Y_{S_1} = 0$.
  - If $Z_{S_1}^{(1)} < M$, $Y_{S_1} = Z_{S_1}^{(1)}$.

From Y_{S_1}, the second maintenance action is planned at time $S_2 = S_1 + \sigma_{M-Y_{S_1}}^{(2)} + \tau = S_1 + U_2$,

- If $Y_{S_2}^- > L$ a corrective replacement is performed at time S_2, $Y_{S_2} = 0$.
- If $X_{S_2}^- \leq L$ a preventive maintenance is performed at time S_2 putting the system back to the deterioration level $Y_{S_1} + Z_{U_2}^{(2)}$.
  - If $Y_{S_1} + Z_{U_2}^{(2)} \geq M$, the system is unmaintainable $Y_{S_2} = 0$.
  - If $Y_{S_1} + Z_{U_2}^{(2)} < M$, $Y_{S_2} = Y_{S_1} + Z_{U_2}^{(2)}$.

More generally, assume $S_1, S_2, \ldots, S_n$ and $(Y_t)_{t \leq S_{n-1}}$ to be constructed with $n \geq 2$. Let $U_n = \sigma_{M-Y_{S_{n-1}}}^{(n)} + \tau$ and $S_n = S_{n-1} + U_n$.

- If $Y_{S_n}^- > L$, the system failed in U_n, hence $Y_{S_n} = 0$.
- If $Y_{S_n}^- \leq L$, a preventive maintenance action puts the system back to the deterioration level $Y_{S_{n-1}} + Z_{U_n}^{(n)}$.
  - If $Y_{S_{n-1}} + Z_{U_n}^{(n)} \geq M$, the system is unmaintainable and it is replaced by a new one at S_n, hence $Y_{S_n} = 0$.
  - If $Y_{S_{n-1}} + Z_{U_n}^{(n)} < M$, the system is not replaced at S_n and $Y_{S_n} = Y_{S_{n-1}} + Z_{U_n}^{(n)}$.

For both models, after a maintenance action at time S_n, the future evolution of the maintained system $(Y_t)_{t \geq S_n}$ depends on the past $(Y_t)_{t \leq S_n}$ only through Y_{S_n} and the process $(Y_t)_{t \geq 0}$ appears as a semi-regenerative process with underlying Markov renewal process $(S_n, Y_{S_n})_{n \in \mathbb{N}}$ and inter-arrival times U_n's. The kernel of $(S_n, Y_{S_n})_{n \in \mathbb{N}}$ is

$$\begin{aligned} q_x(ds, dy) &= \mathbb{P}\left(S_1 \in ds, Y_{S_1} \in dy | Y_0 = x\right) \\ &= \mathbb{P}_x\left(S_1 \in ds, Y_{S_1} \in dy\right) \end{aligned}$$

with support $[\tau, +\infty[\times [0, M]$ because $S_1 \geq \tau$ and $Y_{S_1} \in [0, M]$ almost surely. The main point of the study is to compute this Markov renewal kernal for the two models. As a first step for Model I, we have to compute the distribution of (S_1, Z_{S_1}, X_{S_1}), with $Z_{S_1} = (1-\rho) X_{S_1}$ for Model I and $Z_{S_1} = X_{(1-\rho)S_1}$ for Model II. We only provide elements of proofs for Model I. Technical details may be found in (Mercier & Castro 2013) for Model II, which are much more technical than Model I. As $Z_{S_1} = (1-\rho) X_{S_1}$, the first point for Model I simply is to provide the distribution of (S_1, X_{S_1}).

Proposition 1 *The probability distribution function (p.d.f.) of (S_1, X_{S_1}) is:*

$$h^M(s, z) = \iint_{D_{z,M}} f_{s-\tau}(z-y-u)\, f_\tau(y)\ \mu(du)\ dy \tag{2}$$

for all $s > \tau$ and $x \geq M$, where $\mu(ds)$ denotes the Lévy measure given by (1) and where

$$D_{z,M} = \left\{(u, y) \in \mathbb{R}_+^2 : M \leq z - y < M + u\right\}.$$

Proof 1 (sketch of) *It is already known from (Bertoin 1996) that the p.d.f. of (σ_M, X_{σ_M}) is*

$$g^M(t, y) = \mathbf{1}_{\{M \leq y\}} \int_0^{+\infty} \mathbf{1}_{\{y < M+u\}} f_t(y-u)\ \mu(du).$$

Using the fact that $(S_1, X_{S_1}) = (\sigma_M + \tau, X_{\sigma_M + \tau})$ is identically distributed as $\left(\sigma_M^{(1)}, X_{\sigma_M^{(1)}}^{(1)}\right) + \left(\tau, X_\tau^{(2)}\right)$, *the p.d.f. of (S_1, X_{S_1}) is the convolution of g^M and of the distribution of $\left(\tau, X_\tau^{(1)}\right)$, which provides the result.*

Separating according to the three possible cases at time S_1 (CR, PM+PR, PM), the kernel of $(S_n, Y_{S_n})_{n\in\mathbb{N}}$ may be written as:

$$
\begin{aligned}
&q_x(ds, dy)\\
&= \mathbb{P}_x\left(S_1 \in ds, X_{S_1} > L, 0 \in dy\right)\\
&+\mathbb{P}_x\left(S_1 \in ds, X_{S_1} \le L, X_{S_1} \ge \tfrac{M}{1-\rho}, 0 \in dy\right)\\
&+\mathbb{P}_x\left(S_1 \in ds, X_{S_1} \le L, X_{S_1} < \tfrac{M}{1-\rho}, X_{S_1} \in \tfrac{dy}{1-\rho}\right).
\end{aligned}
$$

Given that the system starts from x, we next use the p.d.f. of (S_1, X_{S_1}) provided by (2) with M substituted by $M - x$, and also replace L by $L - x$ in the previous formula. This provides the following result for the Markov renewal kernel.

Proposition 2 *For Model I, we have:*

$$
\begin{aligned}
&q_x(ds, du) \qquad (3)\\
=\;& \delta_0(du)\int_{L-x}^{\infty} h^{M-x}(s,u)du\\
&+\delta_0(du)\mathbf{1}_{\{M-x<(1-\rho)(L-x)\}}\int_{\frac{M-x}{1-\rho}}^{L-x} h^{M-x}(s,u)du\, ds\\
&+\mathbf{1}_{\{0\le u-x<B_x(M,L)\}} h^{M-x}\left(s, \frac{u-x}{1-\rho}\right)\frac{du}{1-\rho}ds,
\end{aligned}
$$

for all $s > \tau$ and $u < M$, where $B_x(M, L)$ is given by

$$B_x(M, L) = \min((L - x)(1 - \rho), M - x),$$

and $h^M(s, x)$ is provided by (2).

For Model II, the corresponding result from (Mercier & Castro 2013) is:

$$
\begin{aligned}
&q_x(ds, dy) \qquad (4)\\
=\;& \mathbf{1}_{\{y\le M\}}\int_{M-x}^{L-x} u^{M-x}(s, y-x, v)\ dv\ dy\\
&+\delta_0(dy)\int_{M-x}^{L-x} dz\int_{M-x}^{z} u^{M-x}(s, y, z)\ dy\\
&+\delta_0(dy)\int_{L-x}^{+\infty}\int_0^{z} u^{M-x}(s, w, z)\ dw\ dz
\end{aligned}
$$

where

$$
\begin{aligned}
&u^M(s, u, v) = f_{\rho s}(v - u)\cdot \qquad (5)\\
&\int_0^M f_{s-\tau}(x)dx\int_{M-x}^{\infty} f_{\tau-\rho s}(u-t-x)\mu(dt),
\end{aligned}
$$

for $\tau < s < \tau/\rho$ and $M < u < v$, and

$$
\begin{aligned}
&u^M(s, u, v) = f_{(1-\rho)s}(u)\cdot \qquad (6)\\
&\int_M^{\infty} f_\tau(v-w)dw\int_{w-M}^{\infty} f_{\rho s-\tau}(w-u-t)\mu(dt),
\end{aligned}
$$

for $s > \tau/\rho$ and $u < M < v$.

4 RELIABILITY MEASURES

Using the kernel of the Markov renewal process $(S_n, Y_{S_n})_n$ for both repair models, Markov renewal equations are given for some reliability measures and for a cost function.

4.1 *Transient Availability*

Let $A_x(t)$ be the probability that the system is working at time t given $Y_0 = x$ with $x \in [0, M]$:

$$A_x(t) = \mathbb{P}_x(Y_t < L).$$

In case $t \le \tau$, it is easy to show that

$$A_x(t) = F_t(L - x),$$

for both repair models. For $t > \tau$, one has

Theorem 1 *For both models, the availability function fulfills the following Markov renewal equation*

$$A_x(t) = G_x(t) + \int_\tau^t\int_0^M A_y(t-s)q_x(ds, dy),$$

for all $t > \tau$, $x \in [0, M]$ where $G_x(t)$ is given by

$$G_x(t) = \int_0^{M-x} f_{t-\tau}(y)F_\tau(L-x-y)dy \qquad (7)$$

and where $q_x(ds, dy)$ is given by (3) for Model I and by (4) for Model II.

Proof 2 (sketch of) *For $t > \tau$, one classically separates according to whether the first maintenance time is greater or smaller than t:*

$$A_x(t) = \mathbb{P}_x\left(Y_t < L, S_1 > t\right) + \mathbb{P}_x\left(Y_t < L, S_1 \le t\right).$$

The first term is

$$
\begin{aligned}
&\mathbb{P}_x\left(Y_t < L, S_1 > t\right)\\
&= \mathbb{P}_x\left(X_t < L, \sigma_M > t-\tau\right)\\
&= \mathbb{P}_x\left(X_t < L, X_{t-\tau} < M\right)\\
&= \mathbb{P}\left(X_t < L - x, X_{t-\tau} < M - x\right)
\end{aligned}
$$

This provides the first term based on $X_t = X_\tau + (X_t - X_\tau)$ and the independence of X_τ and $X_t - X_\tau$.

As for the second term, by conditionning by the past of the process up to time S_1 and using the Markov property at time S_1, we get:

$$\begin{aligned}\mathbb{P}_x\left(Y_t<L,S_1\le t\right) &= \mathbb{E}_x\left[\mathbf{1}_{\{S_1\le t\}}\mathbb{E}\left(\mathbf{1}_{\{Y_t<L\}}|\mathcal{F}_{S_1}\right)\right]\\ &= \mathbb{E}_x\left[\mathbf{1}_{\{S_1\le t\}}A_{Y_{S_1}}(t-S_1)\right]\\ &= \int_\tau^t\int_0^M A_y(t-s)q_x(ds,dy)\end{aligned}$$

4.2 *Transient Reliability*

Let $R_x(t)$ be the probability that the system does not stop functioning in $(0,t)$ given $Y_0=x$ with $x\in[0,M]$, that is,

$$R_x(t)=\mathbb{P}_x(T>t),\quad t\ge 0,$$

where T is the time to failure of the maintained system. In case $t\le\tau$, it is easy to show that

$$R_x(t)=F_t(L-x),$$

for both repair models. We next envision the case where $t>\tau$.

Theorem 2 *The reliability function fulfills the following Markov renewal equation*

$$R_x(t)=G_x(t)+\int_\tau^t\int_0^M R_y(t-s)\nu_x(ds,dy),$$

for all $t>\tau$, $x\in[0,M]$ where $G_x(t)$ is given by (7) and $\nu_x(ds,dy)$ denotes the kernel of the Markov renewal process $(S_n,Y_{S_n})_{n\in\mathbb{N}}$ restricted to the operating states.

For Model I, this kernel is:

$$\begin{aligned}&\nu_x(ds,du)=\\ &\delta_0(du)\mathbf{1}_{\{M-x<(1-\rho)(L-x)\}}\int_{\frac{M-x}{1-\rho}}^{L-x}h^{M-x}(s,u)du\,ds\\ &+\mathbf{1}_{\{0\le u-x<B_x(M,L)\}}h^{M-x}\left(s,\frac{u-x}{1-\rho}\right)\frac{du}{1-\rho}\,ds.\end{aligned}$$

For Model II, it is:

$$\begin{aligned}&\nu_x(ds,dy)\\ &= \mathbf{1}_{\{y\le M\}}\int_{M-x}^{L-x}u^{M-x}\left(s,y-x,v\right)\,dv\,dy\\ &+\delta_0(dy)\int_{M-x}^{L-x}dz\int_{M-x}^{z}u^{M-x}\left(s,y,z\right)\,dy\end{aligned}$$

where u^M is provided by $(5,6)$.

4.3 *Transient expected cost*

Let $c_x(t)$ be the mean cumulated cost on $]0,t]$ given that $Y_0=x$ with $x\in[0,M]$, that is,

$$c_x(t)=\mathbb{E}_x\left[C\left(]0,t]\right)\right],$$

where $C\left(]0,t]\right)$ denotes the maintenance cost in $]0,t]$. We calculate $c_x(t)$ for both repair models considering the following sequence of costs: c_{CR} corrective replacement cost, c_{PR} preventive replacement cost, c_R repair cost and c_d downtime cost per unit time.

For $t\le\tau$ and for the two models of repair,

$$\begin{aligned}c_x(t) &= c_d\int_0^t\mathbb{P}(t-u>\sigma_{L-x})du\\ &= c_d\int_0^t\bar{F}_{t-u}(L-x)du.\end{aligned}$$

For $t>\tau$, we have a similar Markov renewal equation as for the previous indicators:

$$c_x(t)=H_x(t)+\int_\tau^t\int_0^M c_y(t-s)q_x(ds,dy),$$

for $t>\tau$, $x\in[0,M]$, where $H_x(t)$ is a complicated function, with expression provided in Theorem 3 of (Mercier & Castro 2013), and where $q_x(ds,dy)$ is given by (3) for Model I and by (4) for Model II.

5 NUMERICAL EXAMPLES

In order to illustrate the analytical results, several numerical examples based on Monte Carlo simulations are here considered.

Firstly, let $\alpha=1.5$ and $\beta=3$ be the parameters of the gamma process. The system is assumed to be new at time 0, that is $Y_0=0$. The failure threshold is $L=10$. The induced approximated expected time to exceed level 10 is $\mathbb{E}\left(\sigma_L\right)\simeq 20.37$ time units. The maintenance efficiency is provided by $\rho=0.5$. The costs associated with the different maintenance actions are $c_{CR}=100$ monetary units (m.u.), $c_{PR}=60$ m.u., $c_R=5$ m.u. and $c_d=2$ m.u. per time unit. Let $\tau=10$ time units be the delay time to start the maintenance tasks.

Figure 1 shows the transient availability of the system versus M at time $t=75$ for the two models. Figure 1 has been performed with 100 values from 0 to 10 and 40000 realizations in each point. As we can check by inspection, similar values are obtained for the transient availability for both models of repair.

Anagolously, Figure 2 shows the expected cost rate versus M at time $t=75$ for both repair models. A maximum difference of 19.1295 m.u is found in the expected cost for the two models. For this simulation,

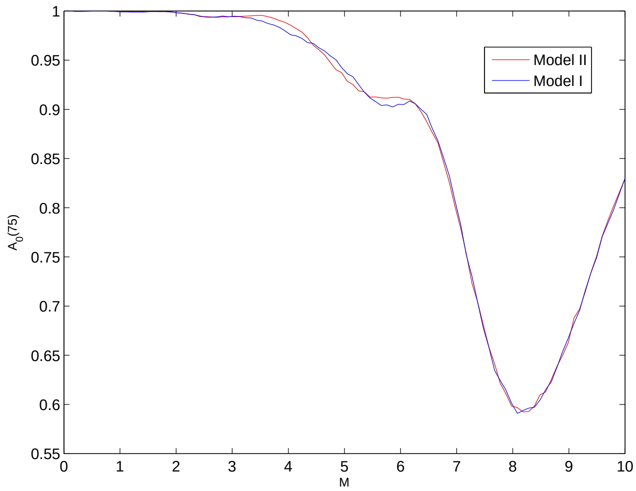

Figure 1: Availability versus M at time $t = 75$, $\rho = 0.5$ and $\tau = 10$

the proportion of the different maintenance actions are the following. For Model I: 7.69% repairs, 56.96% corrective replacements and 35.35% preventive replacements. For Model II, 5.69% repairs, 56.93% corrective replacements and 35.43% preventive replacements.

For the same data set and increasing the maintenance efficiency ($\rho = 0.75$), Figures 3 and 4 show the availability versus M at time $t = 75$ and the expected cost versus M at time $t = 75$ respectively for both repair models. There are slight differences for the values of the transient availability of the two repair models. For the expected cost, we get a maximum difference of 10.14 m.u. For this data set, the proportion of maintenance actions are the following. For Model I: 30.74% repairs, 55.34% corrective replacements and 13.91% preventive replacements. For Model II, 30.68% repairs, 55.42% corrective replacements and 13.90% preventive replacements.

The delay time to perform the maintenance tasks is decreased ($\tau = 2$) maintaining the same efficiency ($\rho = 0.75$) and the initial data set ($\alpha = 1.5$, $\beta = 3$, $L = 10$, $c_{CR} = 100$ m.u., $c_{PR} = 60$ m.u., $c_R = 5$ m.u. and $c_d = 2$ m.u. per time units). Figures 5 and 6 show the transient availability and the expected cost rate at time $t = 75$. As before, similar values for the availability transient are obtained for the two repair models. For the expected cost, a maximum difference of 255.7646 m.u is found for low values of M. For this data set and this simulation, the proportion of maintenance actions are the following. For Model I: 74.63% repairs, 4.64% corrective replacements and 20.73% preventive replacements. For model II, 69.94% repairs, 4.60% corrective replacements and 25.45% preventive replacements.

Finally, some figures showing the transient reliability for both models of repair are given. As in the previous examples, the gamma process parameters and the failure threshold are given by $\alpha = 1.5$, $\beta = 3$ and

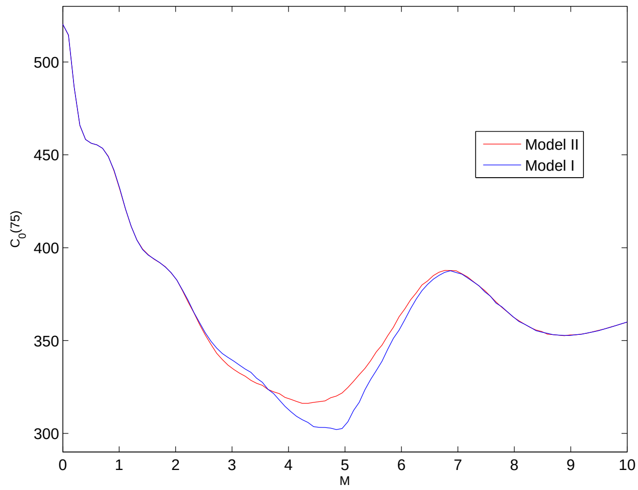

Figure 2: Expected cost versus M at time $t = 75$ with $\rho = 0.5$ and $\tau = 10$

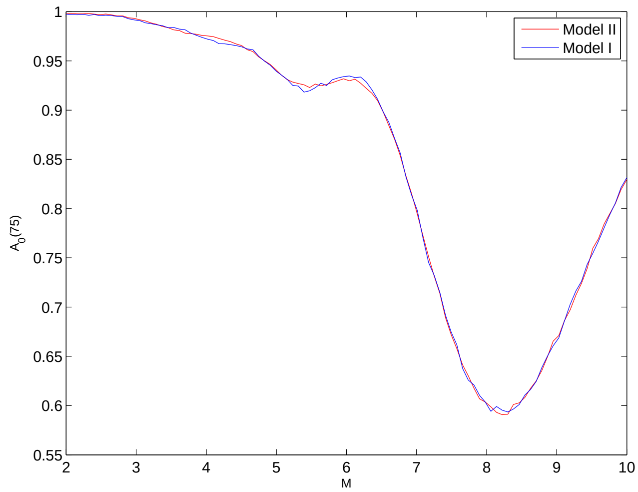

Figure 3: Availability versus M at time $t = 75$ with $\rho = 0.75$ and $\tau = 10$

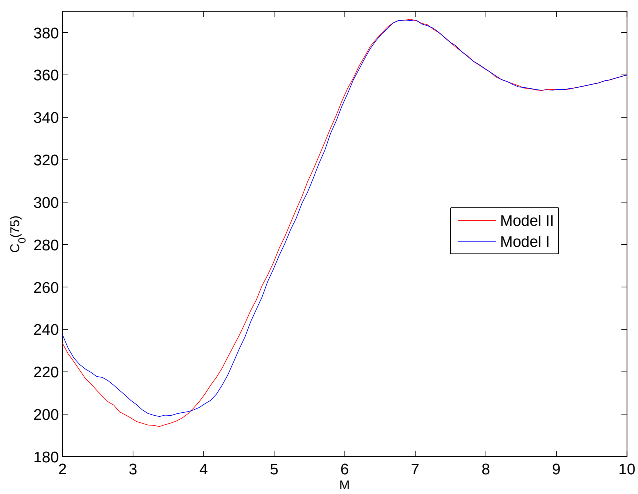

Figure 4: Expected cost versus M at time $t = 75$ with $\rho = 0.75$ and $\tau = 10$

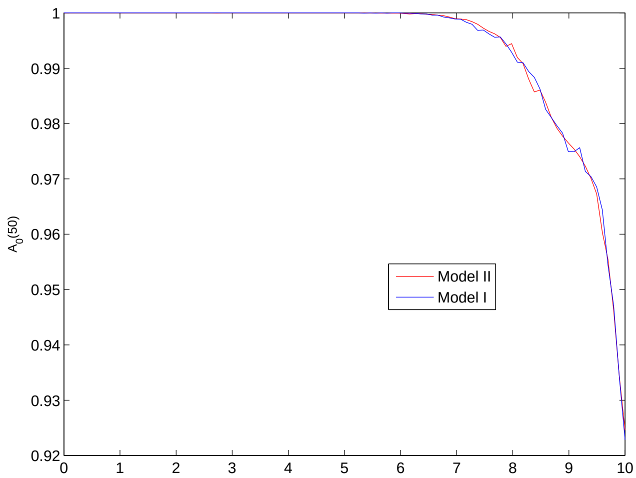

Figure 5: Availability versus M at time $t = 75$ with $\rho = 0.75$ and $\tau = 2$

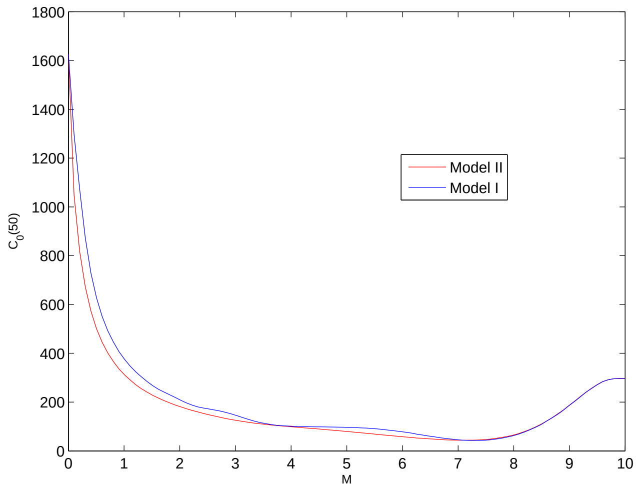

Figure 6: Expected cost versus M at time $t = 75$ with $\rho = 0.75$ and $\tau = 2$

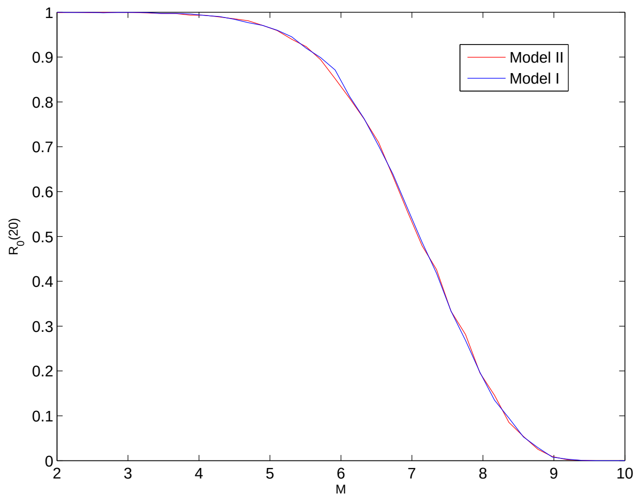

Figure 7: Reliability versus M at time $t = 20$

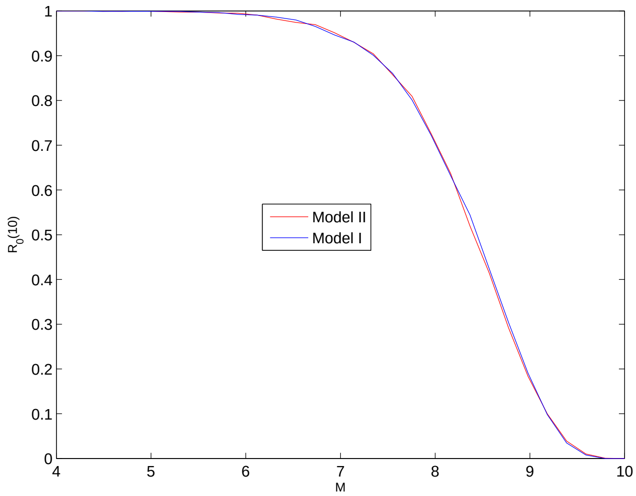

Figure 8: Reliability versus M at time $t = 10$

$L = 10$ respectively. For a maintenance efficiency represented by $\rho = 0.75$ with a delay time to perform the maintenance given by $\tau = 5$ units of time, Figure 7 shows the reliability at time $t = 20$ versus M. Decreasing the delay time to perform the maintenance ($\tau = 3$), Figure 8 shows the reliability at time $t = 10$ for both types of repair.

6 CONCLUSIONS

In this paper, the reliability analysis of a system subject to a continuous degradation modelled as a gamma process is analyzed. Different maintenance actions (preventive replacements, corrective replacements and repairs) are performed. The repairs are imperfect and they are developed under two models: Model I that reduces the degradation of the system and Model II that reduces the age of the system. For both models, the fuctioning of the system is described through a semi-regenerative process. We obtain that the transient availability, reliability and expected cost fulfill Markov renewal equations. Numerical examples based on Monte-Carlo simulations are given. These numerical examples show that transient availability and reliability values are similar for the two repair models and the differences between them are found in the expected cost. The numerical examples of this paper have been performed using Monte-Carlo simulation due to the complexity of the Markov renewal equations. A future extension of this paper is to analyze the transient behavior of the system using a recursive numerical scheme based on the Markov renewal equations provided in the paper. This should allow numerical computations to be quicker and to perform comparisons between the two models on a larger scale.

REFERENCES

Bérenguer, C., A. Grall, L. Dieulle, & M. Roussignol (2003). Maintenance policy for a continuously monitored deteriorating system. *Probability in the Engineering and Informational Sciences 17*(2), 235–250.

Bertoin, J. (1996). *Lévy Processes, Volume 121 of Cambridge Tracts in Mathematics*. Cambridge University Press.

Castanier, B., C. Bérenguer, & A. Grall (2003). A sequential condition-based repair/replacement policy with non-periodic inspections for a system subject to continuous wear. *Applied Stochastic Models in Business and Industry 19*(4), 327–347.

Doyen, L. & O. Gaudoin (2004). Classes of imperfect repair models based on reduction of failure intensity or virtual age. *Reliability Engineering & System Safety 84*(1), 45–56.

Grall, A., C. Bérenguer, & L. Dieulle (2002). A condition-based maintenance policy for stochastically deteriorating systems. *Reliability Engineering & System Safety 76*(2), 167–180.

Huynh, K., A. Barros, C. Bérenguer, & I. Castro (2011). A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events. *Reliability Engineering & System Safety 96*(4), 497–508.

Mercier, S. & I. Castro (2013). On the modelling of imperfect repairs for a continuously monitored gama wear process through reduction. *Journal of Applied Probability 50*(4).

Newby, M. & C. Barker (2006). A bivariate process model for maintenance and inspection planning. *International Journal of Pressure Vessels and Piping 83*(4), 270–275.

Nicolai, R., J. Frenk, & R. Dekker (2009). Modelling and optimizing imperfect maintenance of coatings on steel structures. *Structural Safety 31*(3), 234–244.

Singpurwalla, N. (1995). Survival in dynamic environments. *Statistical Science 10*(1), 86–103.